ARITHMETIC REVIEW

(Includes Basic Statistics)

$$1, \sqrt{4}, 3, 2^2, \ldots$$

Arithmetic Review

Contents

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Introduction

A lack of arithmetic skills can really hold a student back in more advanced courses. It may be tempting to think that you can always just use a calculator, but it isn't that simple. Algebra problems usually require easy calculations only so students can focus on the principles. Someone who needs to grab a calculator to compute 2.5 x 10 will be at a disadvantage. Many problems require students to quickly spot common factors, like the fact that both 63 and 49 are divisible by 7. It is definitely worth learning those times tables, and there are apps and online games that can make it more fun. Inexpensive or home-made manipulatives are a big help for addition and subtraction, and understanding fractions.

While most students can learn to memorize times tables and do simple mental math, there are exceptions. If you have tried and tried but failed to make any progress it may be time to move on. A student who can understand the concepts but cannot do calculations can still advance to higher math with the help of a calculator. Get a good calculator in a color that the student likes, and make sure that it can do fractions. Use it as needed, and to check answers.

Arithmetic Skills Summary

This is a summary only; detailed explanations are provided afterwards.

1. **I can count up to 100**
2. **I understand the size of numbers up to 100 (40 is bigger than 29)**
3. **I understand that numbers are organized in groups of 10**
4. **I can add numbers that have a sum less than 10**
5. **I can subtract numbers that have a sum less than 10**
6. **I know all the number pairs that add up to 10**
   0 + 10, 1 + 9, 2 + 8, 3 + 7, 4 + 6, 5 + 5
7. **I can add two single digit numbers that have a sum greater than 10**
   8 + 6 = ? First do 8 + 2 to get to 10, then add 4 more to get 14
8. **I can subtract a single digit number from a two digit number less than or equal to 20**
   15 – 7 = ? First do 15 – 5 to get to 10, then subtract 2 more to get 8
9. **I can add a single digit number to a two digit number in my head**
   45 + 8 = ? First do 45 + 5 = 50, then add 3 more to get 53
10. **I can subtract a single digit number from a two digit number up to 99**
    85 – 7 = ? First do 85 – 5 to get to 80, then subtract 2 more to get 78
11. **I understand place value (tens, hundreds, thousands, ten thousands)**
12. **I can add two digit numbers with a sum less than 100 in my head**
    48 + 36 = ? Add the tens first: 40 + 30 = 70. Then add the ones: 8 + 6 = 14. Now you have 70 and 14, which is 84.
13. **I can subtract two digit numbers in my head**
    62 – 49 = ? First do 62 – 40 because that is easy. Now you have 22. Then subtract the 9: 22 – 2 = 20, and 20 – 7 = 13.
14. **I can add numbers on paper using carrying**
15. **I can subtract numbers on paper using borrowing**
16. **I understand multiplication**
17. **I can multiply single digit numbers**
18. **I know how to quickly multiply any number by 10**
    Add a zero at the end. 24 x 10 = 240
19. **I know my times tables up to the 10 times table**
20. **I can do division involving the numbers in the first 10 times tables**
21. **I can multiply a two-digit number by a single digit number in my head**
    15 x 6 = ? First do 10 times 6 because it is easiest, then do 5 times 6. 60 + 30 = 90.
22. **I can multiply a two-digit number by a single digit number on paper**

23. I can multiply two two-digit numbers on paper

24. I can multiply any two numbers on paper

25. I know how to quickly multiply any number by 100 or 1000

Add 2 or 3 zeros to the end of the number. 35 x 100 = 3500. 35 x 1000 = 35000.

26. I know the difference between odd and even numbers

Odd numbers cannot be divided by 2, and even numbers can.

27. I can quickly divide any number by 2

To divide 58 by 2, first divide 50 by 2 to get 25. Next, divide 8 by 2 to get 4. Then add up the results: 25 + 4 = 29.

28. I know basic divisibility rules

Is it divisible by 3? Add up all the digits of the number and see if the result can be divided by 3. Can you divide 3477 by 3? 3 + 4 + 7 + 7 = 21. Because you can divide 21 by 3, 3477 is divisible by 3.

Is it divisible by 4? Just check if the last 2 digits of the number can be divided by 4. 47624 is divisible by 4 because 24 can be divided by 4.

Is it divisible by 5? All numbers that end in a 5 or a 0 can be divided by 5.

Is it divisible by 10? Only numbers that end in a 0 can be divided by 10.

29. I can do long division (See the explanations after this list for why you will still need this skill.) The most common mistake when doing long division is forgetting to put a 0 in the answer if it goes zero times.

30. I know how to check my work after I have done a long division

Remember to add the remainder!

Fractions

31. I understand fractions I can draw them and figure out that 1/4 is bigger than 1/5.

I know that a fraction is really a division. $\frac{6}{3}$ is 6 *divided by* 3, which is 2.

32. I can convert mixed numbers to fractions and the other way around

$2\frac{1}{4} = \frac{9}{4}$ Draw a picture so you can see how many quarters there are.

33. I can convert fractions to equivalent fractions $\frac{1}{3} = \frac{7}{21}$. Always multiply the top and the bottom by the same number, or divide them both by the same number. You'll know you did it right if **cross-multiplying** gives the same answers: 1 x 21 = 3 x 7.

34. I can add and subtract fractions

1 apple + 1 apple = 2 apples, so $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$. You can add or subtract fractions easily if they

are the same size. If they are different sizes you must make them the same first: $\frac{1}{3} - \frac{1}{4} = \frac{4}{12} - \frac{3}{12} = \frac{1}{12}$.

35. I can multiply a fraction by a whole number

2 apples x 5 = 10 apples, so $\frac{2}{3}$ x 5 = $\frac{10}{3}$.

36. I can multiply a fraction by another fraction

Multiplying fractions is easy; you just multiply across. $\frac{3}{5}$ x $\frac{2}{7}$ = $\frac{6}{35}$.

37. I can divide with fractions

To divide by a fraction, flip it around and multiply. $\frac{1}{3} \div \frac{1}{4} = \frac{1}{3}$ x $\frac{4}{1} = \frac{4}{3}$. If one of the numbers is a whole number, just change it to a fraction. $5 \div \frac{1}{2} = \frac{5}{1} \div \frac{1}{2} = \frac{5}{1}$ x $\frac{2}{1} = \frac{10}{1}$.

38. I can find a fractional part of a number

$\frac{1}{4}$ of 20 is ? When you divide 20 into 4 equal parts, one of those parts is 5. Learn to take a shortcut by multiplying: $\frac{1}{4}$ x 20 = $\frac{1}{4}$ x $\frac{20}{1} = \frac{20}{4}$ = 5. This multiplication conveniently causes 20 to be divided by 4 to give us the correct answer of 5. If we want $\frac{3}{4}$ of 20, the multiplication conveniently makes the answer 3 times bigger so we get 15. $\frac{3}{4}$ of 20 is $\frac{3}{4}$ x 20 = 15.

Decimals

39. I understand decimals and decimal place values

0.1 = $\frac{1}{10}$ 0. 01 = $\frac{1}{100}$ 0. 001 = $\frac{1}{1000}$

40. I can add or subtract numbers with decimals

Line up the decimal points, and add some zeros behind the decimal point if you need to.
10.5 – 0.01 = 10.50 – 0.01 = 10.49

41. I can quickly multiply any decimal number by 10, 100 or 1000

Move the decimal point one, two, or three places to the right. 0.015 x 10 = 0.15, 0.015 x 100 = 1.5, 0.015 x 1000 = 15.

42. I can quickly divide any number by 10, 100 or 1000

Move the decimal point one, two, or three places to the left. 15 ÷ 10 = 1.5, 15 ÷ 100 = 0.15, 15 ÷ 1000 = 0.015.

43. I can multiply two decimal numbers

First ignore the decimal points and just multiply. Then count how many *total* places there were behind the decimal point. That should be the number of places behind the decimal point in your answer. 13.42 x 5.113 =? 6888486. The first number had 2 places after the decimal, and the second number had 3 places after the decimal. Put the decimal point in the answer so that there are 5 places after the decimal point. 13.42 x 5.113 = 68.88486.

44. I can divide numbers with decimals

First, multiply both numbers by 10, 100 or 1000 if needed so that the number you are *dividing by* does not have a decimal. 100.53 ÷ 0.4 is the same as 1005.3 ÷ 4. Now start dividing. Once you get to the decimal point in your long division, remember to put a decimal point in your answer. When you have passed the decimal point you can add as many zeros as you need to finish your division. 1005.300 ÷ 4 = 251.325.

45. I can change any fraction to a decimal number

$\frac{1}{4}$ means 1 divided by 4. You can always do the long division: 1 ÷ 4 = 0.25.

Some fractions are easily changed to a decimal fraction: $\frac{1}{4} = \frac{25}{100}$, which is 0.25.

Percent

46. I understand what a percentage is

Percent means per hundred or "hundredths". 5% = $\frac{5}{100}$ 50% = $\frac{50}{100}$

47. I can convert between decimals and percent

When you put a % sign behind a number, you are making it 100 times smaller. To compensate for that, make your decimal number 100 times bigger. 0.04 = 4%, 0.7 = 70%.

48. I can find a percentage of number (what is 15% of 40)

To find a fraction of a number, like $\frac{1}{4}$ of 20, we can just multiply. $\frac{1}{4}$ x 20 = 5. A percentage is a fraction too. 15% of 40 is $\frac{15}{100}$ x 40 = $\frac{600}{100}$ = 6. Calculator: 0.15 x 40 = 6.

49. I can convert a fraction to a percentage

There are two ways to do this. The first way is to realize that a fraction is really a division. $\frac{1}{5}$ is actually 1 divided by 5, so we can just do a long division to get 0.2, which is 20%. Sometimes it is easier to convert the fraction directly: $\frac{1}{5} = \frac{20}{100}$ = 20%.

Numbers

50. I can round numbers, and use estimation to quickly spot an incorrect answer

If the next number is 5 or higher, round up; otherwise round down. 13.5 rounds to 14, while 13.4 rounds to 13. Use rounding to estimate 20.37 x 4.99: the answer should be about 100.

51. I can find the average of several numbers

Add the numbers together, then divide by how many numbers there are. The average of 3, 10 and 11 is 24 ÷ 3 which is 8.

52. I can find a missing number if I know the average of a set of numbers

Suppose you had a grade of 80% on your first test, 70% on the second test, and 90% on the 4th test, and your average is 81%. What grade did you get on the third test? Because you know how the average is calculated, you know that your four grades had to add up to 81 x 4 or 324. The missing grade is 324 – 80 – 70 – 90 or 84%.

53. I know the difference between a prime number and a composite number

A prime number can only be divided by two different numbers: 1 and itself. Prime numbers are 2, 3, 5, 7, 9, 11, 13, …..etc. Composite numbers can be divided by something other than 1 or themselves. The smallest positive composite number is 4.

54. I can break down composite numbers into prime number factors

55. I can find the greatest common factor

Break the numbers down into their prime factors; then see what they have in common. 63 is 3 x 3 x 7 and 90 is 2 x 3 x 3 x 5, so the greatest common factor is 3 x 3 = 9.

56. I can find the least common multiple

14 is 2 x 7and 30 is 2 x 3 x 5. The least common multiple is the number that "contains" both these numbers, which is 2 x 3 x 5 x 7 = 210. You only need one 2, not both.

57. I understand exponents

5^2 means 5 x 5, 10^3 means 10 x 10 x 10, and 2^4 means 2 x 2 x 2 x 2.

58. I understand scientific notation

25,000 = 2.5×10^4 0.0132 = 1.32×10^{-2}

59. I understand parentheses

Things inside parentheses always have priority. 10 – 3 + 4 = 11, but 10 – (3 + 4) = 3.

60. I know the order of operations

Please Excuse My Dear Aunt Sally. (Parentheses, Exponents, Multiplication and Division, Addition and Subtraction. Do division and multiplication in order from left to right. Do addition and subtraction in order from left to right.

61. I know what square roots and cube roots are

The square root of 25, or $\sqrt{25}$ is 5 because 5 x 5 is 25. The cube root of 8, which is written as $\sqrt[3]{8}$, is 2 because 2 x 2 x 2 = 8.

62. I understand negative numbers

63. I can do addition and subtraction with negative numbers

Use a number line, or think of negative numbers as debt

Time and Measurement

64. I can do problems involving time

There are 60 minutes in an hour. If it is 8:45 and you need to know what time it will be in half an hour, first add 15 minutes to get to 9:00. Then add another 15 minutes to get to 9:15. If it is 8:05 and you want to know what time it was 15 minutes ago, subtract 5 minutes first to get to 8 o'clock, then take away another 10 minutes to get 7:50.

65. I can read an analog clock

Analog clocks won't go away for a long time yet. Many show Roman numerals instead of regular numbers. The small hand of the clock indicates the hour, and the big hand shows the minutes. A time like 9:15 is often given as "a quarter past nine" when using a clock with hands. 8:45 can be said as "a quarter to nine".

66. I can read work with basic units of measurement (regular and metric)

Know mile (mi), yard (yd), feet (ft), inches (in), hour (hr), minute (min), second (sec), mile (mi), kilometer (km), meter (m), centimeter (cm), kilogram (kg), gram (g), ton, pound (lb), ounce (oz), liter (l), milliliter (ml). Make sure you have an idea of about how big these are.

67. I can do problems involving speed

If a car takes 2 hours to travel 120 miles, its average speed is 60 miles per hour. (Speed = $\frac{\text{distance}}{\text{time}}$). If you travel at a speed of 60 miles per hour for 2 hours, you have covered a distance of 120 miles (distance = speed x time). If your destination is 120 miles away and you travel at a speed of 60 miles per hour, your trip will take 2 hours (time = $\frac{\text{distance}}{\text{speed}}$).

68. I can find the area and perimeter of a rectangle

To find the area of a rectangle, multiply the length times the width. The perimeter is the distance *all the way around*; don't just add the length and the width!

69. I can find the area of a triangle

The area of a triangle is always half of the area of the rectangle that you can draw around it. Draw pictures to understand how this leads to the formula "one-half the base times the height."

70. I can find the volume of a box

Length times width times height, or the area of the base times the height.

71. I can find the surface area of a box

72. I can understand the scale on a map

73. I can use a coordinate system

Remember that the horizontal axis is called the x-axis, and the vertical axis is called the y-axis. Coordinate points are always inside parentheses. The first number represents the distance along the x-axis, and the second number is the distance along the y-axis.

Statistics and Probability

74. I understand ratios

A ratio is a relationship between two values. The ratio of boys to girls in the Smith family is 2 to 1 (often written as 2 : 1). There are twice as many boys as girls in the Smith Family.

75. I can solve proportions. $\frac{2}{1} = \frac{4}{2}$. If the Smiths have 6 children, there must be 4 boys and 2 girls. *Cross-multiply* to check that the two ratios are the same: 2 x 2 = 1 x 4.

76. I understand basic probability

Dice have 6 numbers on them. The chance that I will roll a 4 is 1 out of 6 or $\frac{1}{6}$. Three of the six numbers on dice are even. The chance that I will roll an even number is 3 out of 6 or $\frac{1}{2}$. Write down all the possible outcomes, and then pick the ones you want.

77. I can I can work with basic combinations

If there are 4 flavors of ice cream and 3 different kinds of cones, you can make 12 different one-scoop ice cream cones.

78. I can read graphs – line graphs, bar graphs and pie graphs

79. I understand random sampling

Every member of the population must have exactly the same chance of being chosen for the sample. Surprisingly small samples can allow you to make accurate predictions about a large population.

80. I can find the mean and the median

The mean is just the average of the numbers. The median is the middle number, or the average of the two numbers in the middle.

81. I can find the mode and the range

The mode is the number that occurs most frequently. The range is the difference between the highest number and the lowest number.

82. I can find the upper and lower quartiles

First find the median. The middle of the numbers on either side of the median are the quartiles. Odd number of data points: the median is the middle number; the quartiles are between two numbers and you must take the average. Even number of data points: the median is between two numbers and you must take the average; the quartiles are the middle numbers of each half.

83. I can draw a box and whisker plot

The upper and lower edges of the box are the quartiles. The median is a line inside the box.

The whiskers extend to the lowest and highest values, but they should be no longer than 1.5 times the length of the box (the interquartile range). Points outside that may be outliers.

Explanations for some important topics are provided below. A thorough review of arithmetic is available here as a complete course: http://www.themathpage.com/ARITH/arithmetic.htm. Another good website is http://www.bbc.co.uk/skillswise/maths

Practice Your Times Tables

No, those times tables never go away. If you don't know them well, it will haunt you all the way through college and beyond! You should know the first 10 times tables really, really well.

It may help you to realize that each times table is just a set of additions. Suppose I know that 5 x 4 is 20, but I can't remember 6 x 4, all I need to do is add another group of 4 to 20 to get 24. That also makes it easy to do 9 x 4, because it is just 4 less than 10 x 4. You can subtract 4 from 40 to find that 9 x 4 = 36.

One of the reasons to extend your knowledge up to the 12 times tables is to help you more easily convert between inches and feet.

If you know the first 10 times tables well, you can also handle something like 6 x 12 by doing it like this: 6 x 10 = 60, and 6 x 2 = 12. 60 + 12 = 72. Always do the part with the 10 first, because it takes less space to store in your short-term memory.

If you only want to learn the first 10 times tables it is still a good idea to memorize that 11 x 11 = 121, and 12 x 12 = 144.

Practice times tables here. Select Multiplication, level 3:

http://www.factmonster.com/math/flashcards.html.

http://www.fun4thebrain.com/multiplication/kaigasmult.html

[If you have access to a TI-83or TI-84 Programmable Graphing Calculator (a rather expensive piece of equipment), you can copy the program below.]

Most of the commands used in this program are available through the PRGM menu. The function randInt() is accessed from the PRB section of the MATH menu. The expressions = and > are found under TEST, above the MATH key. The STO key stores a quantity in the variable you name, and it displays an arrow →. To run the finished program, quit the editor and use the EXEC part of the PRGM menu. Highlight the program, hit ENTER, and then press ENTER again. Note that the numbers used for the multiplications are adjustable, so you can remove the 11 and 12, and even the 2 times tables if you want.

```
Program:  TIMES
:Lbl 10
:randInt(3,12)→B
:If B=10
:Then
:Goto 10
:Else
:randInt(2,11)→C
:If C>9
:Then
:C+1→C
:Goto 15
:Else
:Goto 15
:Lbl 15
:Disp B
:Disp "            X"
:Disp C
:Prompt A
:If A=B*C
:Then
:Disp "          : )"
:Goto 10
```

```
:Else
:Disp "          : ("
:Goto 15
:End
```

Long Division

Now that calculators are so readily available, it is tempting to skip long division altogether. Unfortunately you will still need this skill in algebra for problems like $2x^2 + 6x - 5$ divided by $x + 2$. You could learn it when you reach that point, but you'll be busy dealing with other things.

[How to do Long Division (Step by Step) | 1-Digit Divisors - YouTube]

Long division works by first considering the largest part of a number and dividing it up. After that you deal with what is left over. To check your work, multiply your answer by the number you are dividing by, and add the remainder. 587 ÷ 5 = 117 remainder 2. Check: 5 x 117 = 585, and 585 + 2 = 587. By using decimal points you could continue to divide and get an answer of 117.4, or you can divide the remainder to end up with a fraction. You could then write your answer as $117\frac{2}{5}$. That is the sort of thing you will also be doing later on in algebra.

Estimation

It is easy to make a mistake when you are doing math, or maybe you just hit the wrong button on your calculator. A quick estimate can often help you see if an answer is just way too big or way too small to be correct. Estimation is never an exact mathematical procedure. Your answer will depend on the estimation strategy you use. Try to pick a strategy that seems to be the best fit for the problem.

Simple rounding is the most commonly used way of estimating. We tend to round to the nearest "convenient" number. For example, if you are adding 178 + 512 + 199, it is most convenient to round the numbers to the nearest hundred: 200 + 500 + 200 = 900. The answer should be about 900. If you used a calculator and got 86,203, something obviously went wrong.

Small dollar amounts are often rounded to the nearest whole dollar: 5 x $4.99 is about 5 x $5 which is $25. Due to rounding, which adds a penny to each $4.99, the answer is too large by 5 pennies. You can subtract those back off to get an exact answer of $24.95. A larger dollar value like $803.45 may be more conveniently rounded to $800 rather than to the nearest dollar.

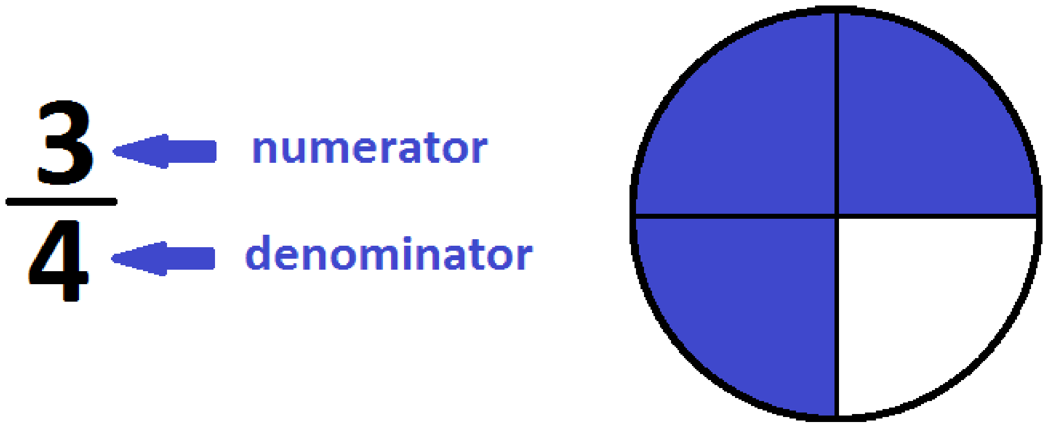
If you are adding a group of numbers that are all fairly close to a certain value, you can round and multiply. For example, 603 + 598 + 597 + 614 + 599 + 601 is about 6 times 600, which would be 3600.

When you are dividing, see if the given number is close to another number that is convenient for your division. To estimate 11 ÷ 6, consider 12 ÷ 6 instead. The answer should be about 2. The actual result will be a little smaller because 11 is not as big as 12.

Front-end estimation in its most basic form considers only the first digit of a number, or the digit of the largest place value of a set of numbers. Retailers know that selling an item for say $8.99 makes it seem cheaper than selling it for $9, even though the difference is only a penny. Consumers just pay more attention to the digit that represents the dollars than they do to the number of cents. Obviously, if you only look at the first digit and ignore the rest of the number, you underestimate the amount, possibly by quite a lot. There are many ways to modify basic front-end estimation to get better results, but those are usually more trouble than just grabbing a calculator.

Fractions

The top part of a fraction is called the numerator, because it gives you the number of pieces. The bottom part is the denominator. The denominator tells you what kind of pieces you have. For the fraction $\frac{3}{4}$, the denominator is 4. This tells you that you have quarters, which are one-fourth pieces. The numerator is 3, so you have 3 quarters.



Here are the 8 steps to understanding fractions:

1. **A fraction is really a division.** $\frac{1}{4}$ means 1 divided by 4, which is one quarter. We can read $\frac{8}{4}$ as "8 quarters", but it also means 8 divided by 4, which is 2.

2. **We can draw fractions to see what is happening.** Pie fractions are easiest to draw. You should always imagine your favorite kind of pie when you are doing this so that you learn to like fractions. Drawing pies helps you understand that 3 wholes is the same as 12 quarters. When you look at the size of the pie pieces, you can see that $\frac{1}{3}$ is bigger than $\frac{1}{4}$, even though 4 is a larger number than 3.

3. **Some fractions are the same size as others.** For example, $\frac{1}{2}$ is the same size as $\frac{2}{4}$, and $\frac{1}{3}$ is the same size as $\frac{2}{6}$. We can see this easily when we are working with pies. We can change the size of fraction pieces by cutting them up, or mathematically by multiplying the top and bottom by the same number: $\frac{1}{2} = \frac{1 \times 4}{2 \times 4} = \frac{4}{8}$. This multiplication makes the top four times as big, and the bottom four times as big, so the ratio doesn't change. We can also combine smaller pieces to make bigger fractions, or divide the top and bottom by the same number. $\frac{4}{8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$.

4. **If you want to add or subtract fractions, the pieces must be the same size.** Otherwise it really doesn't make sense to try to add or subtract. $\frac{1}{2} + \frac{1}{4} = ?$ There are two pieces, but they are both different. That's like trying to add apples and oranges. 1 apple + 1 orange = ?? However, if you call them both fruits you can say that you have 2 fruits. Change $\frac{1}{2} + \frac{1}{4}$ to read

$\frac{2}{4} + \frac{1}{4}$. Now you have 3 pieces, 2 *quarters* and 1 *quarter*. All pieces are the same size so you can add them. The answer is $\frac{3}{4}$. Many times you will have to change the size of both pieces: $\frac{1}{3} + \frac{1}{4}$ = ? Look for a number that both 3 and 4 "go into". That number is 12. Change both fractions to twelfths: $\frac{1}{3} = \frac{4}{12}$, and $\frac{1}{4} = \frac{3}{12}$. Then you can add $\frac{4}{12}$ and $\frac{3}{12}$, which is $\frac{7}{12}$.

5. To get a fraction of a certain amount, multiply it by the fraction. $\frac{1}{4}$ of 200 is $\frac{1}{4} \text{ x } \frac{200}{1} = \frac{200}{4}$, which is 50. If you look closely you can see why it works. To take one-fourth of something, you need to divide it by 4. When you multiply as shown, the 4 ends up on the bottom, automatically causing a division by 4 and giving you the right answer. That also works for multiple parts of something. If you want $\frac{2}{3}$ of 300, you need to divide by 3 to get one third, and then take twice that amount to get two thirds. A simple multiplication like this: $\frac{2}{3} \text{ x } \frac{300}{1}$ accomplishes both things. There is a division by 3, and a multiplication by 2, as required, to give $\frac{600}{3}$ = 200.

6. Multiplying fractions is easy: you just multiply across. $\frac{1}{5} \text{ x } \frac{1}{4} = \frac{1 \text{ x } 1}{5 \text{ x } 4}$, which is $\frac{1}{20}$. To see how that works in real life, think of a large cake. First, we are going to divide the cake in quarters by making long slices:

Next, we cut the cake into 5 parts the other way:

Now when you get one piece, it is one fifth of one quarter of the cake. $\frac{1}{5}$ of something is $\frac{1}{5}$ times that number, so multiply $\frac{1}{5}$ x $\frac{1}{4}$. As you can tell by looking at the cake, there are 20 pieces, so one piece is one twentieth. $\frac{1}{5}$ x $\frac{1}{4}$ must be $\frac{1}{20}$.

You may even get lucky and get two pieces of cake. Then you would get $\frac{2}{5}$ of one quarter of the cake. $\frac{2}{5}$ x $\frac{1}{4}$ = $\frac{2}{20}$, which is one tenth.

If you need to multiply a fraction by a whole number, you can turn it into a fraction:

$2 \text{ x } \frac{1}{5} = \frac{2}{1} \text{ x } \frac{1}{5} = \frac{2}{5}$.

On the other hand, you can just think about what it really means. Two times $\frac{1}{5}$ means that you have two of those pieces: $2 \text{ x } \frac{1}{5} = \frac{2}{5}$.

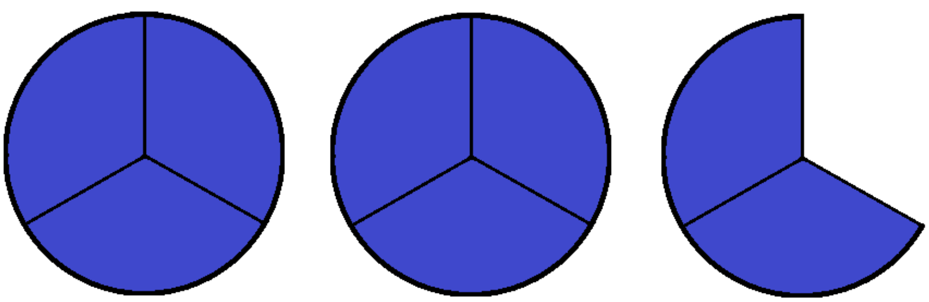
7. When you divide a number by a fraction, it gets bigger. For example, 6 divided by 2 is 3. We can see that this is true because there are 2 groups of 3, or 3 groups of 2, in 6. 6 divided by $\frac{1}{2}$ is 12, because there are 12 halves in the number 6. Just draw some pies to see that. Dividing a number by $\frac{1}{2}$ makes it twice as big. That means dividing by $\frac{1}{2}$ is really the same as multiplying by 2. The trick that goes with that is that when you divide by a fraction, you multiply by its reciprocal (flip the fraction around):

$$6 \div \frac{1}{2} = \frac{6}{1} \div \frac{1}{2} = \frac{6}{1} \times \frac{2}{1} = \frac{12}{1} = 12$$

That works for every division by a fraction:

$$\frac{1}{4} \div \frac{2}{3} = \frac{1}{4} \times \frac{3}{2} = \frac{3}{8}.$$

8. Convert mixed numbers to fractions by cutting up the whole numbers like pies. A mixed number is composed of a whole number and a fraction, like this: $2\frac{2}{3}$. Mixed numbers can always be converted to fractions so you can follow the rules above. To learn how to do that, just start by drawing pies. For $2\frac{2}{3}$, draw three pies and cut the last one into thirds. Then erase one of the pieces so you have two whole pies and two thirds. Next, cut up your whole pies to see how many thirds you have altogether:



There are 8 thirds, so $2\frac{2}{3} = \frac{8}{3}$. Once you have done that quite a few times, you'll know how it works and you won't have to draw pictures anymore. See if you can figure out how many quarters are in $3\frac{1}{4}$ pies. A fraction that has a larger numerator than its denominator is called an improper fraction. To change such a fraction to a mixed number, take the pieces and make whole pies out of them. If you have $\frac{13}{4}$, you can make 3 whole pies. Each of those pies takes 4 quarters to make, so that uses up 12 pieces. Now you have 1 quarter piece left over: $\frac{13}{4} = 3\frac{1}{4}$.

For addition and subtraction it is often easier to leave the mixed numbers and work with them directly. More on that later in this section.

Fraction Movies

Equivalent Fractions: http://www.youtube.com/watch?v=U2ovEuEUxXQ

Mixed Numbers and Improper Fractions: http://www.youtube.com/watch?v=1xuf6ZKF1_I

Fractions Resource

This website has a complete explanation of how to work with fractions. Make sure to look at all the different fraction topics: http://www.mathsisfun.com/fractions.html

Fractions Quizzes

http://www.thatquiz.org/tq-e/math/fractions/reduce/
http://www.quiz-tree.com/Fractions_Practice_main.html

Fractions Game

This game allows you to play with pizzas and fractions. Read the order and select the size of the pizza first. Then you can add the toppings. Next click on "send" to get the money for the pizza. There doesn't seem to be a way of fixing a mistake though, so be careful.

http://www.mrnussbaum.com/tonyfraction.htm

Adding and Subtracting Mixed Numbers

This example shows how to add $5\frac{1}{4} + 3\frac{4}{5}$ without having to change the mixed numbers to fractions:

$$5\frac{1}{4} + 3\frac{4}{5} =$$

Add the whole numbers first, and then worry about the fractions:

$$5 + 3 + \frac{1}{4} + \frac{4}{5} = 8 + \frac{1}{4} + \frac{4}{5}$$

Now look for a common denominator so you can add the fractions. We pick 20 because both 4 and 5 go into 20:

$\frac{1}{4} = \frac{5}{20}$ and $\frac{4}{5} = \frac{16}{20}$

Rewrite the problem:

$$8 + \frac{5}{20} + \frac{16}{20} = 8 + \frac{21}{20}$$

$\frac{21}{20}$ is an **improper fraction** because the numerator is bigger than the denominator. Change the improper fraction to a mixed number:

$$\frac{21}{20} = 1\frac{1}{20}$$

Rewrite again:

$$8 + 1\frac{1}{20} = 9\frac{1}{20}$$

We can also subtract mixed numbers. There are two ways to do that. You could start like this:

$$5\frac{1}{3} - 2\frac{1}{2} = 5\frac{2}{6} - 2\frac{3}{6}$$

Next, because $\frac{2}{6}$ is smaller than $\frac{3}{6}$ you need to look at your $5\,\frac{2}{6}$ pies and cut up one of the whole pies into sixths. Once you cut up one pie you have 4 left: $5\,\frac{2}{6}$ turns into $4\,\frac{8}{6}$. Now it is possible to subtract: $4\,\frac{8}{6} - 2\,\frac{3}{6} = 2\,\frac{5}{6}$.

On the other hand, you could take $5\,\frac{2}{6} - 2\,\frac{3}{6}$ and subtract the whole numbers first:

$$5\,\frac{2}{6} - 2\,\frac{3}{6} = 3\,\frac{2}{6} - \frac{3}{6}$$

I like to start this way because it makes the problem seem smaller. Then you would still have to cut up one of your whole pies. $3\,\frac{2}{6}$ is the same as $2\,\frac{8}{6}$:

$$3\,\frac{2}{6} - \frac{3}{6} = 2\,\frac{8}{6} - \frac{3}{6} = 2\,\frac{5}{6}$$

Here is a video showing how to subtract mixed numbers:

http://www.youtube.com/watch?v=tVrelLu6K6k

Estimating with Fractions

How can you tell if a fraction is bigger than $\frac{1}{2}$ or smaller? Well, first you should know how to tell if something is equal to one half. Let's look at some examples.

$\frac{2}{4} = \frac{1}{2}$

$\frac{3}{6} = \frac{1}{2}$

$\frac{4}{8} = \frac{1}{2}$

$\frac{5}{10} = \frac{1}{2}$

You know this because for each one of these fractions you can divide the top and bottom by the same number to change the fraction to one half. Also, you should be able to spot that all of these fractions are $\frac{1}{2}$ because **the top number is exactly half of the bottom number**. When you look at a fraction like $\frac{15}{30}$, you can see that the top number is half of the bottom number, so this fraction equals $\frac{1}{2}$.

When there is an odd number in the denominator, the top number can't really be half of the bottom number. You can have $\frac{2}{5}$ or $\frac{3}{5}$, but you can't make $\frac{1}{2}$ out of fifths. To get exactly one half, you would have to write $\frac{2\frac{1}{2}}{5}$, and we don't usually do that. Of course, whatever you want to write for your personal use is fine. I either write it or imagine it so I can tell where the half-way point is. Then I know that $\frac{2}{5}$ is smaller than a half, and $\frac{3}{5}$ is bigger than a half.

Knowing whether something is bigger or smaller than a half can be useful. Students sometimes try to add fractions in the same way as multiplying them, by just adding across. That causes them to claim that $\frac{1}{3} + \frac{1}{4}$ is equal to $\frac{2}{7}$. If you have a good idea of about how big fractions are, you can see that this doesn't look right. One third is a little bigger than a quarter, and if you add two quarters you get a half. The answer should be a little bigger than a half. $\frac{2}{7}$ on the other hand is definitely smaller than a half, because exactly $\frac{1}{2}$ would be $\frac{3\frac{1}{2}}{7}$. Using estimation, we see that the answer was not correct.

It is also a good idea to know when you are close to a whole. $\frac{11}{12}$ is almost 1, since only a small piece, $\frac{1}{12}$, is required to make it a whole. When a fraction is almost 1, you can round it to 1 and get a reasonable estimate.

You may be asked to round fractions to the nearest half. This means figuring out if the fraction is closer to 0, $\frac{1}{2}$, or 1. As an example, let's round $\frac{2}{9}$ to the nearest half. It would take $\frac{4\frac{1}{2}}{9}$ to make one half. The number 2 is closer to 0 than it is to $4\frac{1}{2}$, because to get to from 2 to $4\frac{1}{2}$ you have to add $2\frac{1}{2}$, but to get from 2 to 0 you only have to subtract 2. We round $\frac{2}{9}$ to 0, because it will give a better estimate than rounding it to $\frac{1}{2}$. If you end up right in the middle between 0 and $\frac{1}{2}$, or exactly between $\frac{1}{2}$ and 1, you will likely be expected to round up, just as you would for regular numbers.

When all of the numbers in a calculation are fractions or very small mixed numbers, it makes sense to round to the nearest half. If you are dealing with larger mixed numbers, you may want to round to the nearest whole number. The small error caused by rounding $15\frac{5}{9}$ to 16 is acceptable in many situations.

Decimals

Converting Fractions to Decimals: http://www.youtube.com/watch?v=Gn2pdkvdbGQ

Assignment

Create your own fractions problem and use a calculator to check your answer, as explained below:

Because a fraction is really a division, you can use a calculator to check your answers to fraction problems. For example, $\frac{3}{4} - \frac{5}{8} = \frac{1}{8}$. To check this on a calculator, we need to type in the problem: 3 ÷ 4 – 5 ÷ 8 = . Math has rules for the order in which things are done. Division and multiplication get priority over addition and subtraction. If your calculator is able to correct for **order of operations** you will get an answer of 0.125. 1 ÷ 8 is also 0.125. Some calculators will do 3 ÷ 4 – 5 ÷ 8 just how it is printed, dividing 3 by 4 first, subtracting 5, and then dividing the answer by 8. If you don't get what you expect use your calculator to do things separately: 3 ÷ 4 = 0.75 and 5 ÷ 8 = 0.625. 0.75 – 0.625 – 0.125.

Decimals Practice

http://www.aaamath.com/plc51b-placevalues.html

Easy Multiplication and Division by 10, 100, 1000

The ten times table is very easy to learn, because you just add a zero to the number you are multiplying. 5 x 10 = 50. This works because adding the zero moves the number 5 from the ones place to the tens place. By adding the zero you make the number 10 times bigger. You can do that to any whole number: 347 x 10 = 3470. To multiply by 100, just add two zeros: 2 x 100 = 200. Multiplication by 1000 adds 3 zeros, and so on.

To do division by 10, you take away a zero. 50 ÷ 10 = 5. To divide by 100 you take away two zeros: 2000 ÷ 100 = 20. We can also divide this number by 1000 by taking away three zeros: 2000 ÷ 1000 = 2.

Unfortunately not all numbers have zeros to take away. Also, sometimes there is a decimal point in a number and we cannot just add a zero to multiply by 10. 6.3 x 10 = ? If we add a zero at the end of 6.3, we don't get anywhere. 6.30 is the same as 6.3. Now we have to start thinking of all multiplications and divisions by 10, 100 and 1000 as moving a decimal point.

Let's go back to 5 x 10. The number 5 actually has a decimal point, but we don't usually write it. 5 is the same as 5.0. When we add a zero to multiply by 10, we can do that by moving the decimal point one place to the right. 5.0 x 10 = 50. To multiply by 100, we move the decimal point two places to the right. First write 5 as 5.00. Then move the decimal point two places: 5.00 x 100 = 500.

6.3 x 10 = ? Here we already have a decimal point, and all we have to do is move it one place to the right. 6.3 x 10 = 63. Be careful not to move the decimal point too far. 2.45 x 10 = 24.5. Each time you move the decimal point to the right one place, you make the number 10 times bigger. To multiply by 100 you move the decimal point two places to the right: 2.45 x 100 = 245. 2.45 x 1000 is the same as 2.450 x 1000. Move the decimal point three places to the right to make the number a thousand times bigger. 2.45 x 1000 = 2450.

Now you might guess how we can divide any number by 10, even if it does not have a zero at the end. 73 ÷ 10 = 7.3. Just move the decimal point one place to the left. Remember that when you divide by 10 the number becomes smaller. Think about what you are doing so that you don't accidentally move the decimal point the wrong way. To divide by 100, move the decimal point two places to the left: 7.3 ÷ 100 = 0.73. If you have to divide by 1000, move the decimal point three places to the left: 7.3 ÷ 1000 = .073. That's all there is to it.

See How it Works

To do multiplication or division by powers of 10, you move the decimal point. That is actually an illusion. The decimal point always stays in the same place, and the numbers move!

In our number system, the decimal point sits between the ones place and the tenths place. If you move it, it still sits between the ones place and the tenths place, so what you have really done is move the numbers:

15.43

The 1 is in the tens place, 5 is in the ones place, 4 is in the tenths place, and 3 is in the hundredths place.

154.3

The 5 is now in the tens place, 4 in the ones place, and 3 is in the tenths place. In fact, all of the digits have moved up one place value. This makes the whole number 10 times bigger.

Practice

If you need some practice you can generate your own worksheets, with solutions:

http://www.dr-mikes-math-games-for-kids.com/multiplying-powers-of-ten.html

Percentages

A percent is a one-hundredths part. 1% = $\frac{1}{100}$. A fraction is really a division, so you can divide 1 by 100 to get 0.01. Any percentage can be converted to a decimal number:

4% = $\frac{4}{100}$ = 0.04 and 60% = $\frac{60}{100}$ = 0.6.

It is also possible to have more than 100%. 200% means $\frac{200}{100}$.

http://www.mathgoodies.com/lessons/vol4/meaning_percent.html

If you answer 18 out of 20 questions on a test correctly, you can determine your percentage grade by writing that as a fraction: $\frac{18}{20}$. You can use long division or grab a calculator to get the answer 0.9. 0.9 is $\frac{9}{10}$ or $\frac{90}{100}$, so that would be 90%. Because I picked some convenient numbers here, you can also use equivalent fractions: $\frac{18}{20} = \frac{18 \text{ x } 5}{20 \text{ x } 5} = \frac{90}{100}$. For some tests not every question counts equally. Your teacher may assign a different number of points to each question. In this case you need to add up your points, and find what percentage of the total possible points that is. For example, if your teacher grades your test and gives you 36 points out of a possible total of 50, you take the fraction $\frac{36}{50}$ and convert that to $\frac{72}{100}$ by multiplying the top and the bottom by 2. Your test score is 72%.

Questions that ask you to find a percentage may be worded in different ways, such as "6 is what percent of 300?" To get the percentage, just write it as a fraction: $\frac{6}{300}$. This fraction is equivalent to $\frac{2}{100}$, or you can use a calculator to get 0.02. The answer is 2%. This answer makes sense because 1%, or one-hundredths part, of 300 would be 3. If 1% of 300 is 3, then 2% should be 6.

To find a fraction or percent of a number you need to use multiplication:

$\frac{1}{5}$ of 200 is $\frac{1}{5}$ times 200: $\frac{1}{5} \text{ x } \frac{200}{1} = \frac{200}{5}$ = 40. Notice that this multiplication by $\frac{1}{5}$ conveniently causes a division by 5 so we end up with a fifth part of 200. We can also use this multiplication trick with percentages because percentages are fractions. 1 divided by 5 is 0.2, or 20%. To find 20% of 200 we can multiply $\frac{20}{100}$ x 200, or 0.2 x 200. The answer is still 40.

Stores often offer discounts. If a shirt that normally costs $40 is on sale at 10% off, then 10% of $40 will be subtracted from the cost of the shirt. It is not so difficult to find 10% of something, because 10% means $\frac{10}{100}$, which is $\frac{1}{10}$. To get one tenth of 40, we multiply:

$\frac{1}{10}$ x 40 = $\frac{1}{10}$ x $\frac{40}{1}$ = $\frac{40}{10}$.

To divide 40 by 10 you move the decimal point one place: 40. → 4.0 → 4. $4 will be subtracted from the price of the shirt before you pay for it. $40 - $4 = $36. In this case it is easy to calculate the final price. When the numbers are a bit harder you may want to use a calculator. Suppose the shirt costs $37.50, and it is on sale at 10% off. One fast way to find the final price on your calculator is to realize that a 10% discount means that you will actually be paying 90% of the cost of the shirt. To get 90% of $37.50, you multiply: 90% x 37.50 = 0.9 x 37.50 = 33.75. The shirt will actually cost $33.75.

When you eat at a restaurant you are expected to give your waiter or waitress a tip. If the service was good, that tip is supposed to be at least 15%. Some restaurants conveniently print the suggested amount on the bill, but sometimes you have to calculate it yourself. When you have a calculator handy, you can just multiply. Suppose your total bill is $28. Multiply 15% x 28, which is the same as 0.15 x 28. If you don't have a calculator, the easiest thing to do is to find 10% first. To get 10% of $28, remember that 10% is one tenth, and you can just divide by 10. Move the decimal point one place: 28.00 → 2.800 → 2.80, which is two dollars and eighty cents. If 10% of $28 is $2.80, then 5% should be half of that, or $1.40. The total tip is 10% + 5%: $2.80 + $1.40 = $4.20.

Convert Fractions to Percentages

http://www.mathgoodies.com/lessons/vol4/fractions_to_percents.html

Converting Decimals to Percentages: http://www.youtube.com/watch?v=RvtdJnYFNhc

Finding the Average

When Jim gets home after being out for Halloween, he has more candy than his little brother who wasn't feeling well and had to go home early. Jim tells his brother to put all his candies on the table. Then he adds his own. Next, he divides the pile of candies evenly, so that both he and his brother have the same amount of candy.

What Jim did was very nice, and it is also a good way to find the average of two numbers. Just add the two numbers, and then divide the total by 2. For example, the average of 8 and 12 is

$\frac{8+12}{2} = \frac{20}{2} = 10$

The number 10 is exactly in the middle between 8 and 12.

To find the average of 3 numbers, add them up and divide by 3. The average of 4 numbers is found by adding them up and dividing by 4, and so on.

Prime Factors

Prime numbers are those numbers that are only divisible by two separate numbers. They can be divided by 1 and by themselves. This means that 1 is not a prime number, because it can only be divided by one number rather than two. The number 2 is prime because it can be divided by 1 and 2.

The first few prime numbers are 2, 3, 5, 7, 11, and 13.

Numbers that are not prime can be split up into prime factors. It works like this:

Find the prime factors of 84.

The smallest prime number is 2, and 84 can be divided by 2: 84 = 2 x 42.

42 can also be divided by 2: 42 = 2 x 21

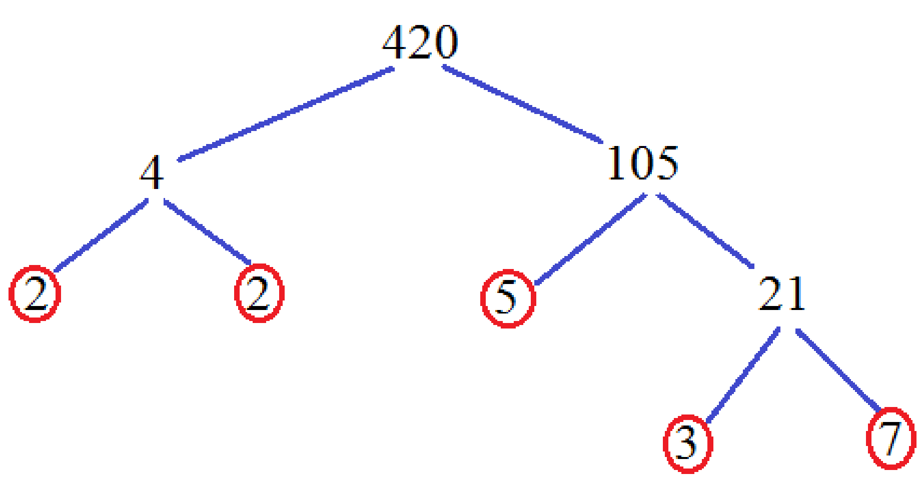
The next smallest prime number is 3. 21 can be divided by 3: 21 = 3 x 7

7 is a prime number.

Put it all together: 84 = 2 x 2 x 3 x 7

Find the prime factors of 420.

Let's do this one by the tree method. The numbers spread out like branches of a tree. You can just divide without worrying about prime numbers. Here I started by dividing 420 by 4, but I could have divided by 2 first, or by 7, or by 10, or anything that works. The end result is always the same - just try it yourself to see that you get the same answer.



Circle your prime factors when you are done and put them in order:

420 = 2 x 2 x 3 x 5 x 7

The Greatest Common Factor

The greatest common factor (GCF) of two numbers is the largest number that you can divide both these numbers by. For example, the GCF of 24 and 42 is 6. Both numbers can be divided by 6, and there is no larger number that they can both be divided by.

We can use prime factors to find the greatest common factor.

24 = 2 x 2 x 2 x 3
42 = 2 x 3 x 7

What these two numbers have in common is the prime factors 2 and 3. Both numbers can be divided by 2 x 3, so they can be divided by 6.

The Least Common Multiple

The least common multiple (LCM) of two numbers is the smallest number that can be divided by both numbers. For example, the least common multiple of 20 and 12 is the number 60. 60 can be divided by 20, and also by 12. There is no smaller number that can be divided by both 20 and 12.

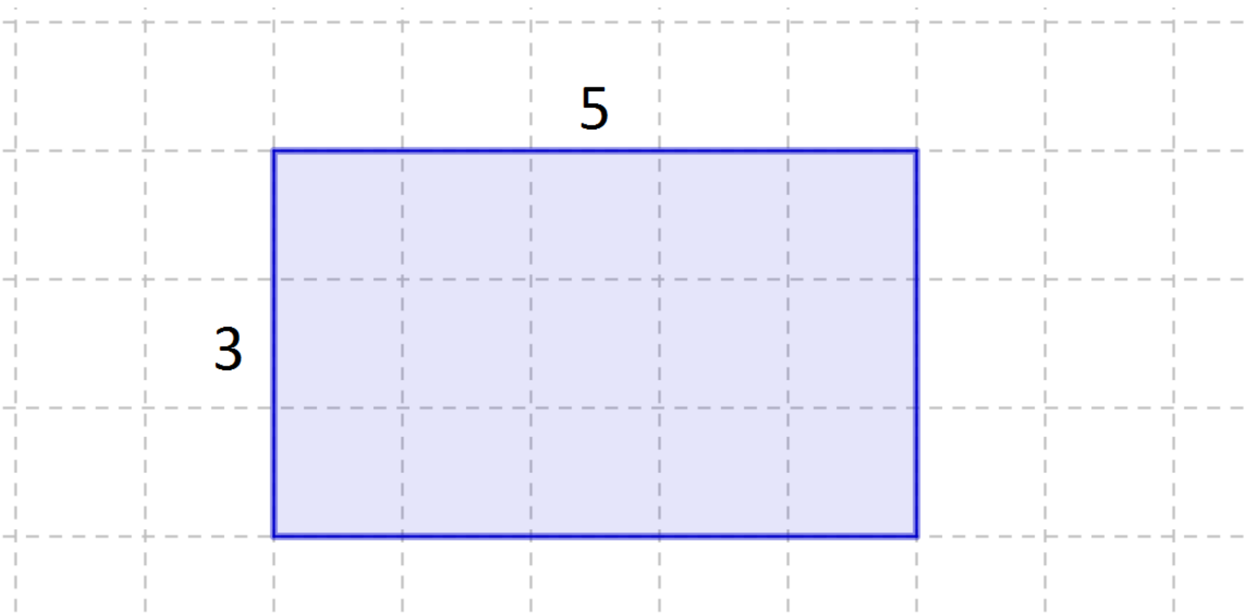
We can find the least common multiple of two numbers by considering the prime factors:

20 = 2 x 2 x 5
12 = 2 x 2 x 3

The least common multiple should contain the prime factors of each number. We will need the factor 2 two times, and also the factors 3 and 5. The least common multiple must be 2 x 2 x 3 x

5 (= 60). This sequence contains 2 x 2 x 5, and it also contains 2 x 2 x 3.

Area and Perimeter



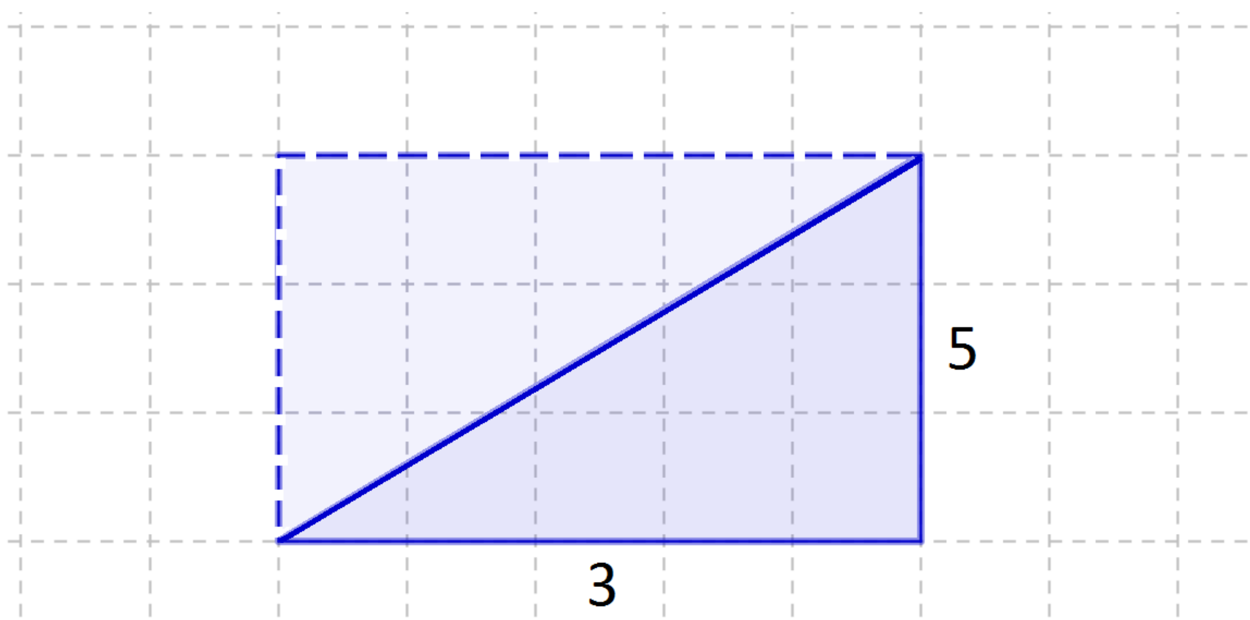
When I think of area, I think of it as the size of the "flat space" on the ground or on a surface. The tricky part is to measure just how big that area is. For the rectangle shown here, the length is 5 units, and the width is 3 units. We can look at the picture to see that the area takes up 15 squares, or 15 square units. This happens because we can fit 5 squares along the length, and 3 squares along the width. That makes 3 rows of 5 squares, which is 15 squares. Area measurements are always stated in square "units", like square feet (ft^2), square inches ($inch^2$ or in^2), square meters(m^2) or whatever units the sides were measured in. To find the area of any rectangle, just think about how many squares can fit inside it. If you multiply the length and the width you will get the answer.

Caution: some people confuse the area with the perimeter. The perimeter is the distance *around* a figure. To find the perimeter, just imagine that you are walking all the way around the figure. For the rectangle shown above, you would walk 5 units, then 3 units, then 5 units, and then 3 units. The total distance is 5 + 3 + 5 + 3 = 16. That is a lot of walking, so you can take a shortcut by only going halfway around, and then multiplying that distance by 2: 5 + 3 = 8, and 8 x 2 = 16. The perimeter is always measured in regular units like inches or centimeters, never in square units like the area.

You may be asked to find the surface area of a box. This is not difficult to do, but it may be a bit hard to imagine when the box is just drawn on paper. It is best to grab a real box to practice on, like a cereal box or a tissue box. Notice that your box is made up of 6 rectangles or squares. All you need to do is find the area of each of those 6 sides, and then add them all up. Luckily, some of the sides are the same. If you look at your box, you will see that the top and the bottom have the same area. So does a side and the opposite side. A cereal box has 3 pairs of surfaces that you have to find the area of.

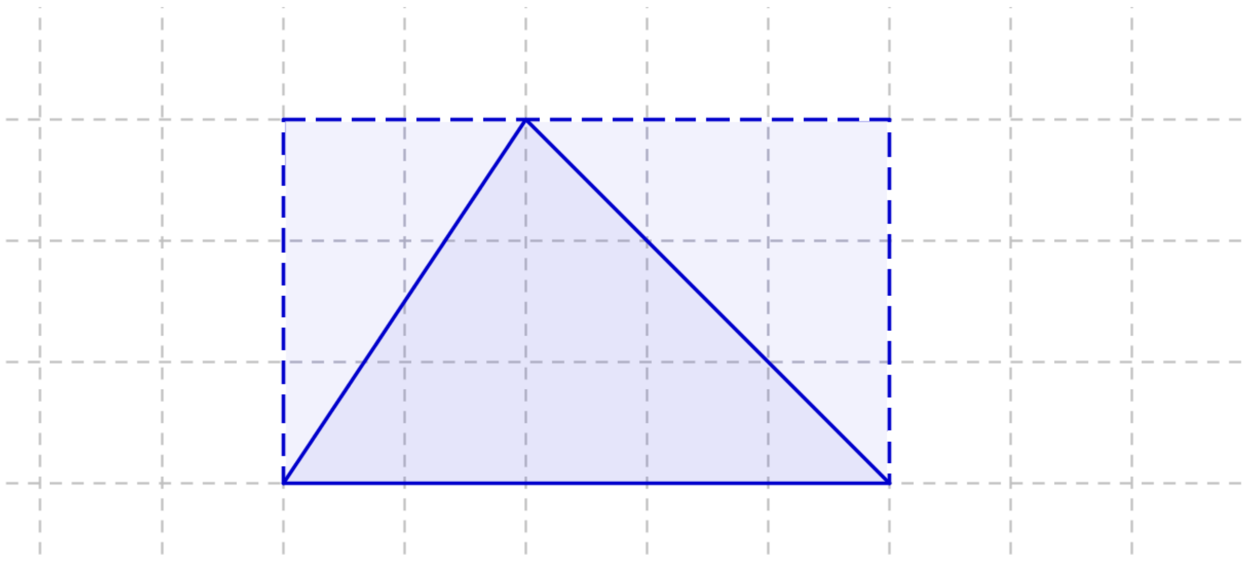
If your box is a perfect cube, all the sides are squares, and they all have the same area. Just find the area of one square and multiply that by 6.

Square units only fit nicely into rectangles or squares. What can you do if you have a triangle?



In this picture, it looks like the area of the bottom triangle should be half of the total area of the rectangle. If you cut out the rectangle and then fold it in half, you will see that both triangles are the same. The area of the bottom triangle is 15 ÷ 2 = $7\frac{1}{2}$ units, and the area of the top triangle is the same.

You can find the area of any triangle by drawing a rectangle around it:



If you cut out this rectangle and fold over the lighter colored parts, you can see that the area of the triangle is exactly half of that of the rectangle. The lighter colored area is the same size as the darker colored area. The bottom, or **base**, of the triangle is 5 units, and its height is 3 units. The area of the triangle is half of 5 x 3, which is $7\frac{1}{2}$ units.

Measurement

To learn about the U.S. system of measurements, go to
http://www.mathsisfun.com/measure/us-standard-units-introduction.html

Exponents

Exponents are small numbers placed to the right and slightly above a number, like this: 5^2. They show how many times the number should be multiplied by itself. 5^2, or five *squared*, means 5 x 5, which is 25. Notice that 5 squared is the area of a square that has sides of length 5. In the same way, 10^2 means 10 x 10 which is 100. Some students try to read 5^2 as "five two". Don't do that, because you may start mixing up 5^2 with two times five which is 10 rather than 25. Always say "five squared", or "five to the second power".

Five *cubed*, or 5^3, means 5 x 5 x 5, which is 125. This is the volume of a cube with sides of length 5.

We don't have special words for exponents higher than 3. 5^4 is just "five to the 4^{th} power". If you have to do multiplications with exponents, just write things out completely:

$2^2 \times 2^3 = 2 \times 2 \;\; \times \;\; 2 \times 2 \times 2 = 2^5$

You can use also use a calculator to find the value of 2^5. For most calculators you would enter 2^5, which then gives you an answer of 32.

Order of Operations

After you learn about exponents, you will need to know that they have high priority.

$4 + 5^2 = ?$

Here the exponent comes first. Do 5 x 5, and then add the 4. Think of the exponent as "stuck to the number", like they are glued together. Once you put an exponent on a number, it changes. 8^2 just means 64.

Another rule about what to do first is that multiplication or division get priority over addition and subtraction. Do everything in order from left to right as written, unless you would be breaking this rule.

And of course people may want to get around these rules when writing math. For this purpose, we have parentheses. We can give priority to anything we want by using them, like this:

$4 + 5^2 = 9$

$(4 + 5)^2 = 81$

While **P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally (Parentheses, Exponents, Multiplication, Division, Addition, Subtraction) is useful to help you remember things, it is a bit misleading. Multiplication and Division have equal priority, and they are done in order from left to right. If the division is written before the multiplication, you do it first. The same goes for addition and subtraction: $10 – 4 + 6 = 12$. If you did the addition first you would get zero.

Roots

Roots are like the reverse of exponents. 5^2 is 25, and the square root of 25 is 5. That is written as $\sqrt{25} = 5$. Square roots can also be seen as the length of the edge of a square. A square that has an area of 25 square inches would have sides that are 5 inches long.

5^3 is 125, and the cube root of 125 is 5. That is written as $\sqrt[3]{125} = 5$. Cube roots can also be seen as the length of the edge of a cube. A cube that has a volume of 125 cubic inches would have sides that are 5 inches long.

Exponents and roots are very important in more advanced mathematics. It is a good idea to read about them online now and do some practice.

Word Problems

Sue went to a bake sale and bought some cupcakes. She paid $6 for a dozen cupcakes. How much did each cupcake cost?

Students who are worried about word problems often panic and try to do the math right away. Actually, **reading comes first and math comes last**. Read the problem carefully and slowly so you can understand what is going on. Imagine what is happening in the problem, or draw a picture. Here you are looking for the **description** part of the problem, like "Sue went to a bake sale and bought some cupcakes."

Make sure you know what it is that you are being asked to find. I often write it down, like this:

cost of 1 cupcake = ?

The person who wrote the problem had to include the **facts** that allow you to solve it. Find those facts. Underline them:

<u>She paid $6 for a dozen cupcakes</u>

or write them down in your own words:

12 cupcakes cost $6

Now start thinking about math.

If 12 cupcakes cost $6, how much does one cupcake cost?

I need to know about 1 cupcake instead of 12. To turn 12 cupcakes into one, I could eat 11, but then what about the money part? If I feel lost here, I might draw 12 circles to represent 12 cupcakes. Then I might look at that for a bit, and maybe split them into two groups. 6 cupcakes is half the amount, so that should cost $3. Let's try dividing both the cupcakes and the cost:

12 cupcakes ÷ 12 = 1 cupcake

$6 ÷ 12 = ?

When you do math, it is often helpful to temporarily remove units like dollars, pounds, miles etc., and just look at the numbers:

$6 \div 12 =$

A division can be written as a fraction:

$6 \div 12 = \frac{6}{12}$

Fractions can be simplified:

$\frac{6}{12} = \frac{1}{2}$

Now put the unit back:

$\frac{1}{2}$ dollars

You may know that half a dollar is 50 cents, or you may first have to think that a whole dollar is 100 cents. You may also want to write your answer as a decimal:

$\frac{1}{2}$ = 0.5, which is the same as 0.50

In dollar notation that would be $0.50

Once you have an answer, make sure that it seems reasonable. If you got an answer of $50 for one cupcake, that would be one really expensive cupcake! You might want to go back and check your work for mistakes. 50 cents for one cupcake seems about right if 12 cupcakes are 6 dollars.

To get more comfortable with word problems, you should try writing your own. You can probably create something much more interesting than what you find in your math book, and you will understand the different parts of word problems better.

Statistics

Experiments involve observations, measurements, and counting. If you observe, measure or count for a long while, and write everything down, you end up with a collection of data. There are many different kinds of data, and many ways to organize and compare them. Statistics is the science of dealing with data and making sense of them. Learn about statistics here: http://www.mathsisfun.com/data/index.html.

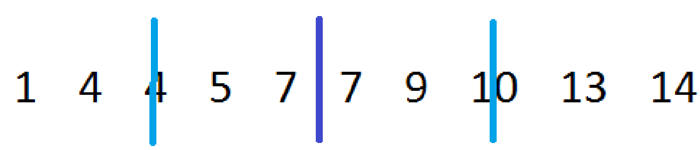
Important terms to learn include **mean**, **median**, **range**, **quartiles**, and **interquartile range**. The mean is the average of the numbers that make up the data. Simply add up the numbers and then divide by how many numbers you have.

To find the **median**, order your numbers from smallest to largest. The median is the middle point of these numbers, which means that it is either the *middle number* or the *average of the two middle numbers*. For the data set 2, 4, 4, 7, 8, the median is 4 because that is the middle number. For 5, 6, 8, 200, the median is 7 because that is the average of 6 and 8.

The median has a definite advantage over the mean if your data contains a few very high or very low values, because such values can really pull the average in one direction. If the *average* grade on a test is 60%, that could mean that most students did really well, but the teacher gave Joe and Maria zeros for cheating. If the *median* grade is 60%, that means that half the students scored at or below 60%, and half the students scored at or above 60%. The median is usually used for house prices, so that a few very expensive homes don't make it seem like the average house is more expensive than it actually is.

The **range** is the distance between the highest value and the lowest value. It shows the "spread" of the data.

The median divides the data in half, and when you divide those halves in half again you get the **quartiles**. This division is done the same way as for the median. If the median is between two numbers, these numbers are included in the half that you divide to get the quartiles:



In the picture above, there are an even number of data points. The median (Q2) is 7, the lower quartile (Q1) is 4 and the upper quartile (Q3) is 10.

If there are an odd number of data points, there is an actual middle number that is the median. Put a line through the middle number and then ignore it as you look for the quartiles:

3 4 6 6 7 9 10 12 12 18 19

Outliers are data points that don't seem to "fit" with the remaining data because their value is too large or too small. Outliers may represent an unusual event or an error in measurement. If you are asked to calculate if a particular point is an outlier, find the interquartile range (the distance between the upper and lower quartile) and multiply it by 1.5. Then add this figure to the number that represents the top quartile and subtract it from the number that represents the bottom quartile. A value outside this range may be considered an outlier. For the data set above, 3, 4, 6, 6, 7, 9, 10, 12, 12, 18, 19, the upper quartile is at 12 and the lower quartile is at 6. The interquartile range is 12 – 6 = 6. 1.5 times the interquartile range is 6 x 1.5 = 9. There should be no numbers that are more than a distance of 9 away from the upper and lower quartiles. The lower quartile is 6, so there is no problem at the low end. The largest number in this data set is 19, which is only 7 more than the upper quartile. If the highest number was 25, it would be an outlier because 25 – 12 = 16, which is definitely more than 9. 12 + 9 = 21, so any number higher than 21 will be considered an outlier

A **boxplot** (box-and-whisker plot) shows the median, the upper and lower quartiles, and the range: http://www.basic-mathematics.com/box-and-whiskers-plot.html. The whiskers may extend to the highest and lowest values in the data set, or unusually high or low values may be marked separately with an asterisk.

The range tells you how much difference there is between the lowest and the highest value in the data. If the range is large, that could be because there is just a single very low or very high value, or it could mean that all of the data points are very spread out. The **mean absolute deviation** measures whether the data is more clustered together or more spread out. To find the mean absolute deviation, first record how far away each data point is from the mean. Because we are considering the distance from the mean, all of these values will be positive or zero (**absolute values** are never negative). Then find the mean (the average) of these values, which is the mean absolute deviation.

Preparing for Algebra

So, you want to get ahead in math and go on to Algebra. You must now complete a series of difficult assignments to prove that you are brave, intelligent, and otherwise worthy of this important subject...... Just kidding 😄! There is nothing special about algebra, and everyone can take it.

Algebra deals with our general understanding of numbers. For example, you know that 3 groups of 5 is the same as 5 groups of 3, and 4 x 6 is the same as 6 x 4. Is it true that 182645553920 x 90784839299021 is the same as 90784839299021 x 182645553920? Why do you think so? Do you really believe that or do you need a calculator? The reason that you don't need a calculator to believe that is that you are making a generalization, based on your experience with smaller numbers. Making generalizations is a natural thing to do, and it is not even uniquely human. Animals make generalizations all the time. Harmless flies that look like bees or wasps don't just scare people; they are usually left alone by predators. My little dog loved to chase cats before we got our own cat. It took some training (and a scratch on the nose), but eventually the dog learned not to bother the cat. When we had a white cat, the little dog would bark at and chase any cat other than white cats. Next we got a black and white cat. Now the dog only chased orange, grey or black cats.

Algebra is the logical next step in the study of numbers. This is why algebra is thousands of years old, and why it is taught in schools. Yet there is no point in forcing yourself to make generalizations about something you don't understand completely in the first place. You need to really understand how to work with numbers before you start algebra. Fortunately, because algebra deals with general things about numbers, there are rarely any hard multiplications or divisions to worry about, and you are often allowed to use a calculator. Most of the numbers you'll come across will be smaller than 100. You can save yourself a lot of time and trouble in algebra if you know your times tables really well. It also really helps to be able to add and subtract numbers less than 20 quickly. Yes, you can always use a calculator, but that takes more time and distracts you from thinking about the algebra in the problem.

When we look at the general idea of division in algebra, we will end up with a lot of fractions. For example, 12 ÷ 6 = $\frac{12}{6}$, which is 2. But if we look at 12 divided by something in general, the fraction just stays there: 12 ÷ some number = $\frac{12}{some\ number}$. Because of that you need to be really comfortable with fractions. This e-book lists some resources in case you need more practice.

Annoying word problems are just as common in algebra as they are in arithmetic. They may actually be more annoying. Look at the following problem: Farmer Smith has 28 cows. He sells 13 cows. Let's call the remaining number of cows 'x'. Then call the original number of cows 'y', and the number of cows that were sold 'z'. How many cows does farmer Smith have now? After looking at this problem, farmer Smith decides to just get rid of all his cows and move to the city to find an easier job. Make sure you have a good grip on word problems before we start sticking some random letters into them. The same strategies still apply. Decide which information in the problem is irrelevant, which information you need, and what you are being asked to find. A lot of practice really helps.

The last thing you'll need to bring into Algebra I is a good understanding of decimals and percentages. If you need more practice use the resources supplied here or find your own.

Take the test below when you are ready.

Algebra Pretest

This test covers only the minimum skills required for students entering Algebra 1. Please work slowly and carefully. Any wrong answer on this test may indicate a problem with your basic skills that you need to correct before moving on to algebra.

Do not use a calculator because you should not need one. (Special needs students who normally rely on calculators may use one for this test but should be able to understand the explanations in the answer key.)

Question 1

$\frac{3}{4}$ = ___ %

Question 2

$\frac{2}{9} \div \frac{3}{4} = \frac{\quad}{\quad}$

Question 3

The width of a rectangle is 5 feet. Its length is 8 feet.

The area of this rectangle is ____ square feet.

The perimeter of this rectangle is ____ feet.

Question 4

Your parents have left you their credit card so you can order pizza for dinner. They have told you to give a 15% tip, rounded to the nearest dollar. When the pizza arrives, the bill is $19.97. How much of a tip do you add to this amount? $ _________

Question 5

A phone which regularly costs $249 is on sale at 5% off. How much does it cost now?

$__________

Question 6

$\frac{2}{5}$ x $\frac{1}{7}$ = $\frac{}{}$

Question 7

3 square feet = ______ square inches

Question 8

Simplify the following fraction so it is in lowest terms:

$\frac{15}{20} = \frac{}{}$

Question 9

If you correctly answer 19 out of 25 questions on a test, your percentage score would be _____ %

Question 10

$\frac{2}{3} - \frac{1}{4} = \frac{}{}$

Question 11

$0.5 = \frac{}{100}$

Question 12

Which is the largest? Choose one answer.

a. $\frac{4}{8}$

b. 0.48

c. 8.4%

d. 4.8%

e. 0.84

Question 13

11 x 0.5 = ______

Question 14

8.4 ÷ 0.5 = _______

Question 15

Sandy takes her three small children to lunch at a local fast food restaurant. The children's meals are $5.99 each. Sandy orders a $9.99 cheeseburger meal for herself. The total bill before tax is $_________

Question 16

Which is the smallest? Choose one answer:

a. $\frac{1}{4}$

b. $\frac{9}{20}$

c. 0.1

d. $\frac{2}{18}$

e. 0.11

Question 17

$3 \times \frac{1}{5} = \frac{\quad}{\quad}$

Question 18

$0.04 = \frac{\quad}{100}$

Question 19

Convert the mixed number to a fraction:

$5\,\frac{1}{8} \;=\; \frac{\quad}{\quad}$

Question 20

Convert to a decimal number: 12% = ______

Question 21

Find the average of these numbers:

4, 11, 13, 8 ___________

Question 22

$3^3 =$

a. 6

b. 27

c. $\frac{1}{9}$

d. 9

e. $\frac{1}{3}$

Question 23

5 + 3 x 2 =

Question 24

2 + $\sqrt{16}$ =

Question 25

Tessa drove at an average speed of 45 miles per hour. How long did it take her to travel a distance of 105 miles? _________________

Algebra Pretest Answers

1. 75%

2. $\frac{8}{27}$

3. Area: 40 sq ft, perimeter: 26 ft

4. Round the bill to $20, tip is $3

5. Ten percent would be $24.90, and 5% is half that or $12.45. Subtract that from the original cost to get $236.55. If you were using a calculator you would consider that the new price is 95% of the original price. Multiply $249 by 0.95.

6. $\frac{2}{35}$

7. 432 square inches

8. $\frac{3}{4}$

9. 76%

10. $\frac{5}{12}$

11. $\frac{50}{100}$ 0.5 is the same as one half, or 5 tenths.

12. e. 0.84 is the only number here that is larger than one half.

13. 5.5 First multiply 0.5 by 10, then multiply it by 1.

14. 16.8 Here you are dividing by 0.5, which is $\frac{1}{2}$. To divide by a fraction, turn it around and multiply: 8.4 x 2 = 16.8.

15. $27.96 Calculate by rounding: 3 x 6 + 10 = 28, then subtract 4 cents.

16. c. 0.1 is $\frac{1}{10}$. Note that $\frac{9}{20}$ is nearly $\frac{10}{20}$, which would be $\frac{1}{2}$

17. $\frac{3}{5}$

18. $\frac{4}{100}$

19. $\frac{41}{8}$

20. 0.12

21. 9 The numbers add up to 36. There are 4 numbers so divide the sum by 4.

22. b. $3^3 = 27$

23. 11 Multiplication is done before addition.

24. 6 The square root of 16 is 4

25. $2\frac{1}{3}$ hours, 2 hours and 20 minutes.